Policy Evaluation - Regression

March 14, 2024

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$$\{Avg_n[Y_i^1|D_i = 1] - Avg_n[Y_i^0|D_i = 0]\} = \{E[Y_i^1|D_i = 1] - E[Y_i^0|D_i = 0]\}$$

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- But...in policy analysis it's often (though not always) impossible to randomize individuals in our sample into treatment?
- How can we attempt to identify causal effects of policy when randomization is infeasible?

Policy Evaluation - Regression Analysis

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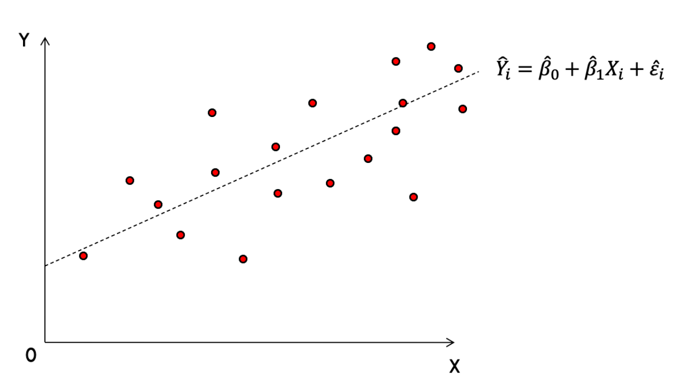
  - And that $ATE_{est} = ATE$.
  - BUT THIS NOT “GUARANTEED” LIKE IT IS WITH RANDOMIZATION AND DEPENDS ON A STRONG (UNTESTABLE) ASSUMPTION.

Policy Evaluation - Regression

- So how does regression actually work?



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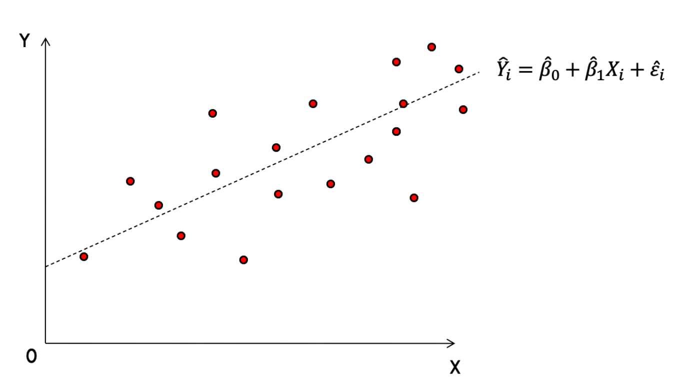

- Note that we have not randomized people into X in this case. We simply observe an individual's value of X in our sample.

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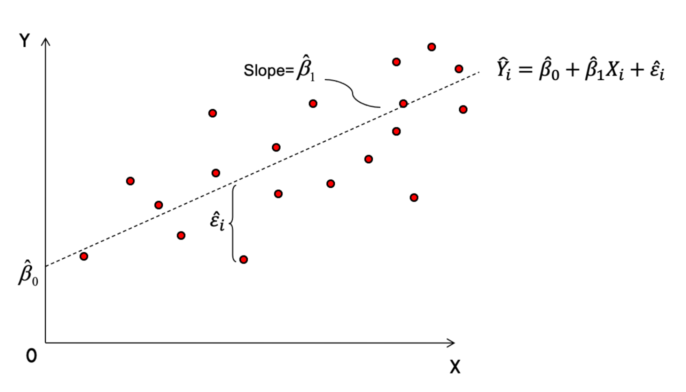

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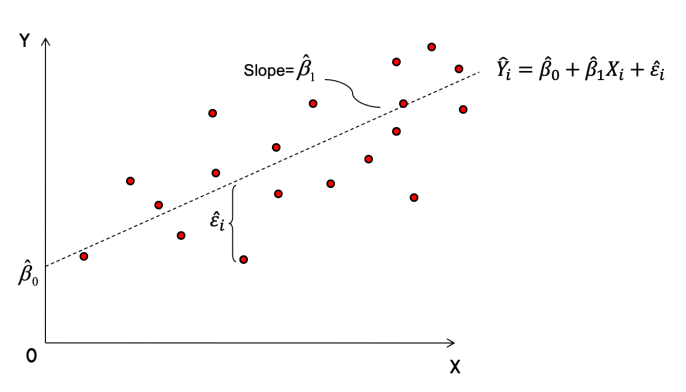


- How does OLS use the regression residuals to fit a trend line to a series of data points?

- Suppose that we estimate a linear regression model of the following form: $y_{it} = \beta_0 + \beta_1 Edu_{it} + \varepsilon_{it}$

  Where y is the number of times that individual *i* visits the emergency department in year *t*

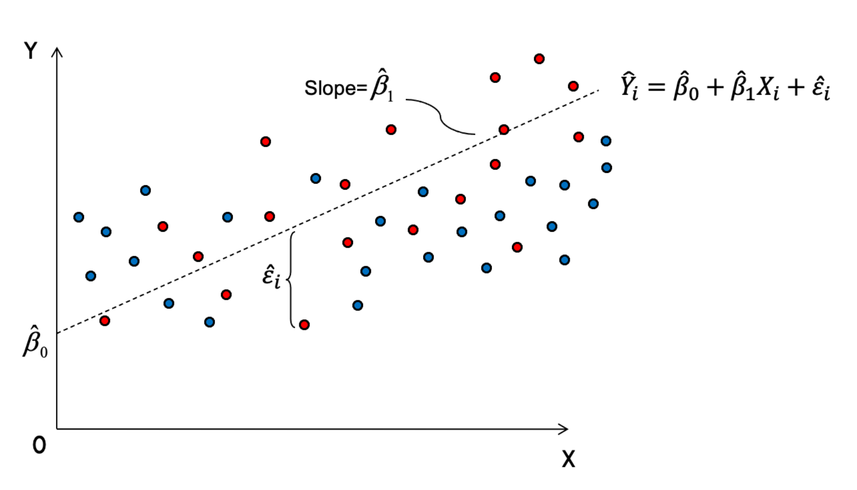
  Edu is individual *i*'s education in years (e.g. a value of 12 represents a high school education, a value of 14 represents a college education, etc.).

  Our coefficient estimates are as follows: $\hat{\beta}_0$=4.0 and $\hat{\beta}_1$=-0.25.

  How do we interpret these coefficient estimates?

Policy Evaluation - Regression

- But suppose the *population* distribution is...



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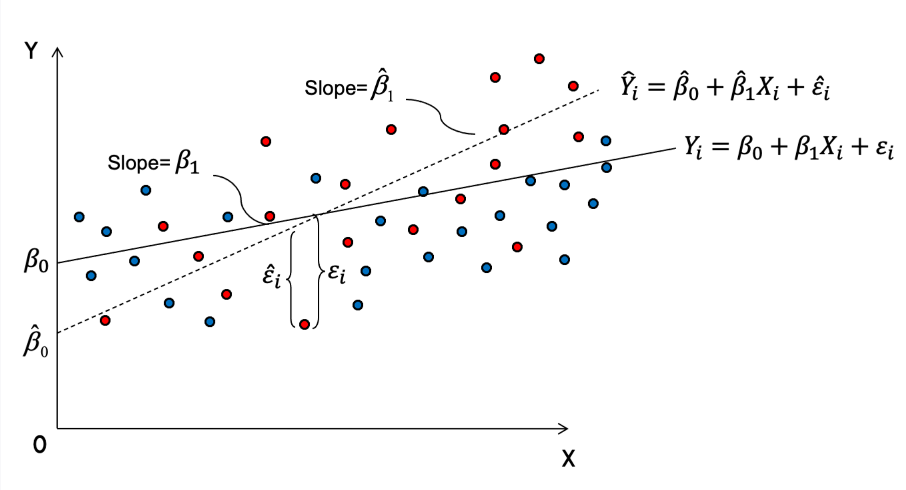

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  - Suppose you estimate the regression model above and find that the coefficient estimate of β_1 has a p-value of 0.115 from a two-sided t-test. What does that mean and what would you conclude about the statistical significance of the relationship between education and emergency department use?

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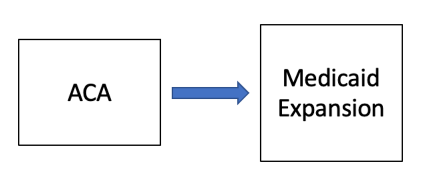
  - In both cases we're just comparing means of treated and control groups from a sample.
- Where these methods begin to diverge is when we add additional covariates to our regression model to "control" for potental confounders.

Policy Evaluation - Regression

- Medicaid Expansion and Mortality



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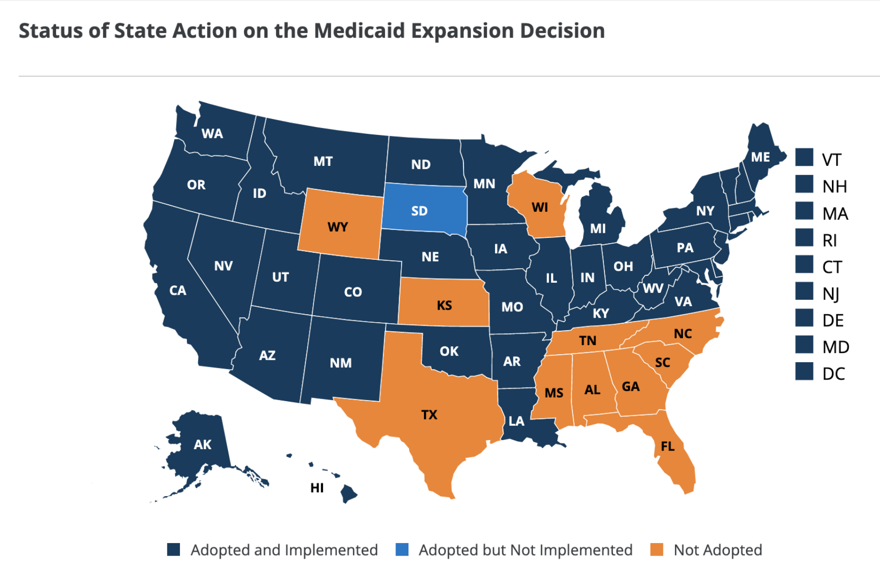

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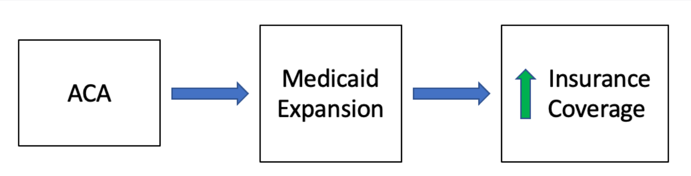

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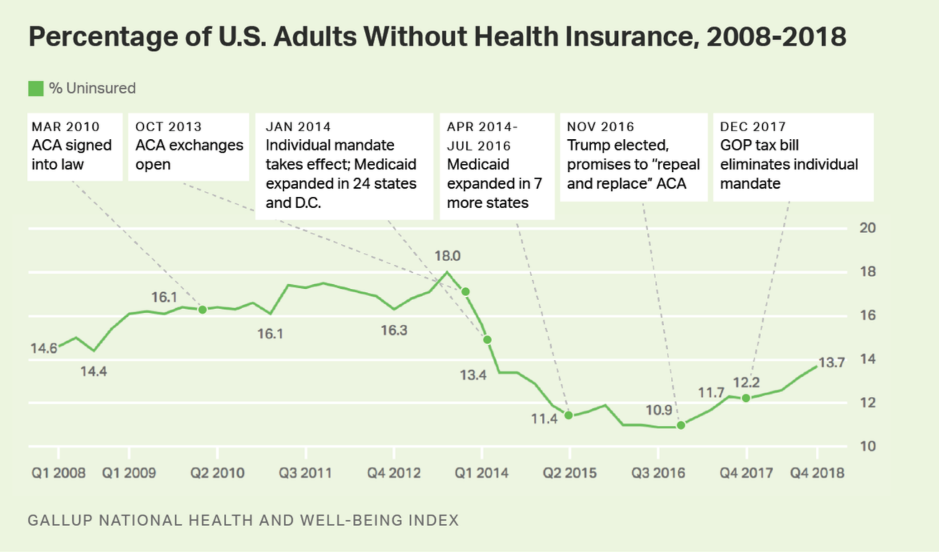

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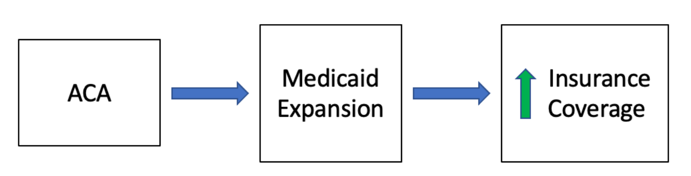

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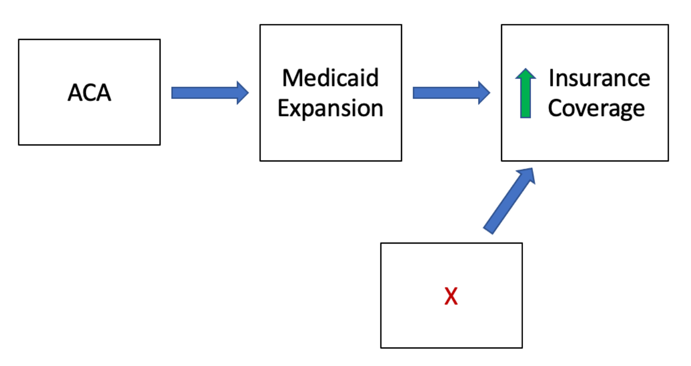

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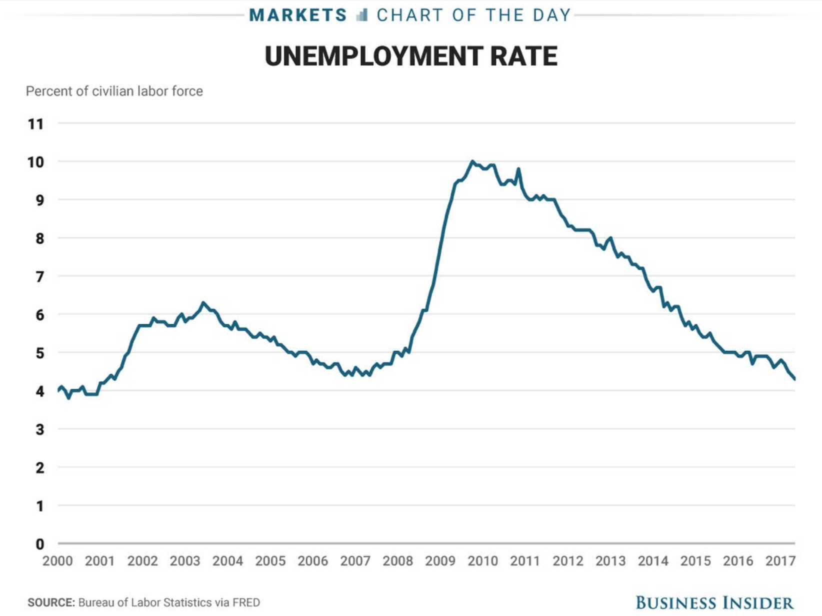

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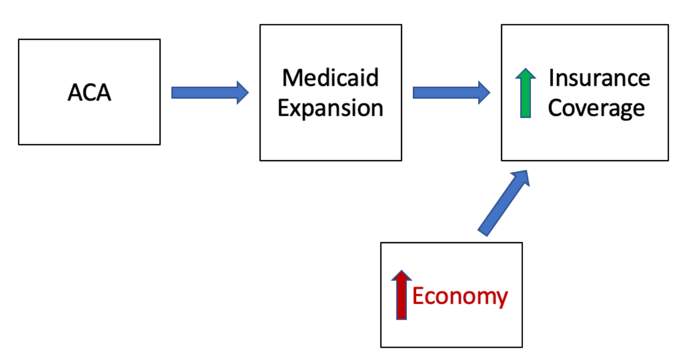

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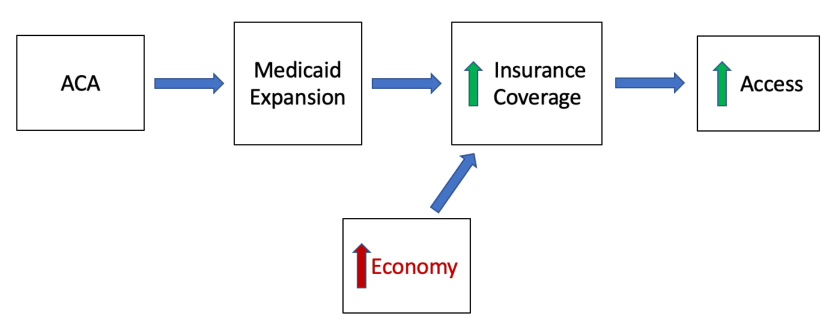

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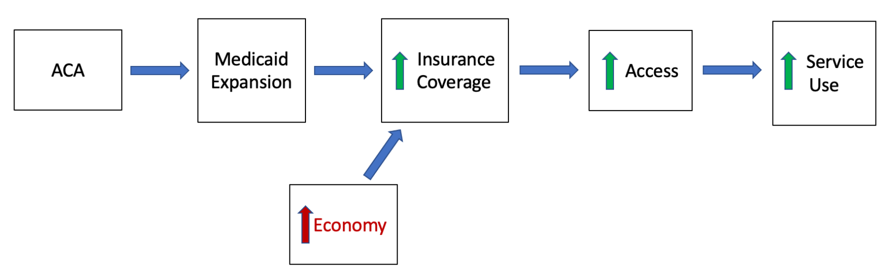

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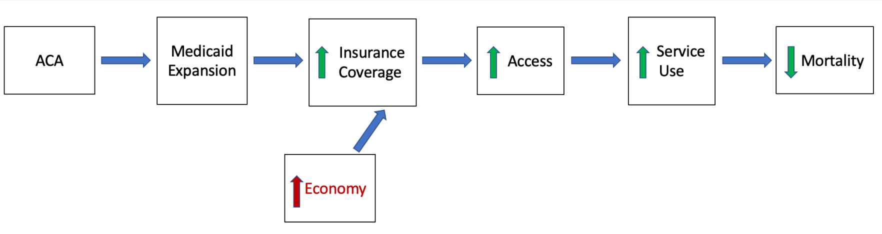

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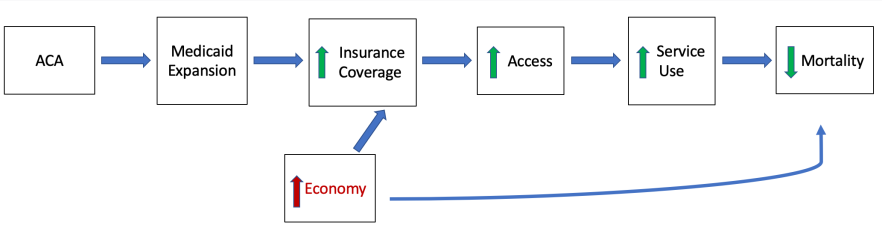

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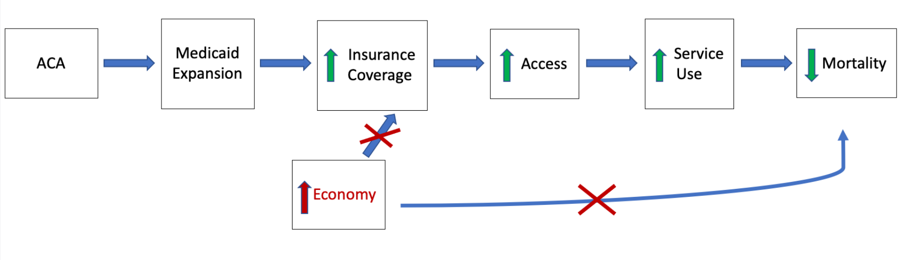

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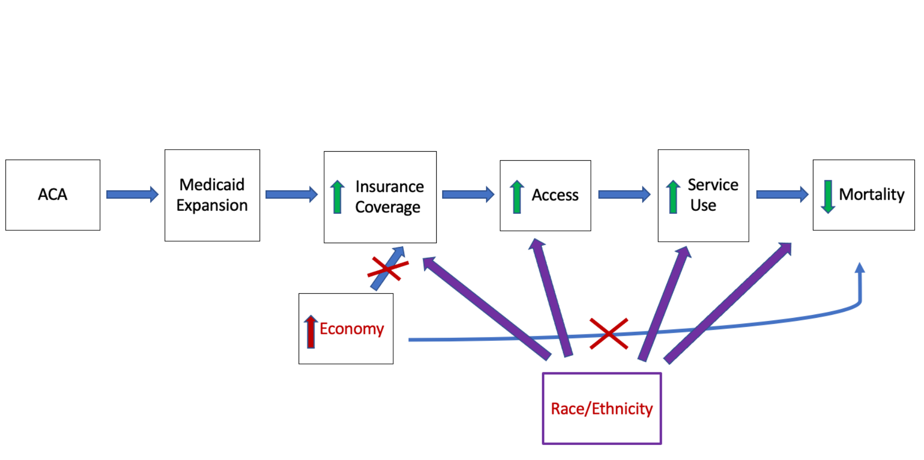

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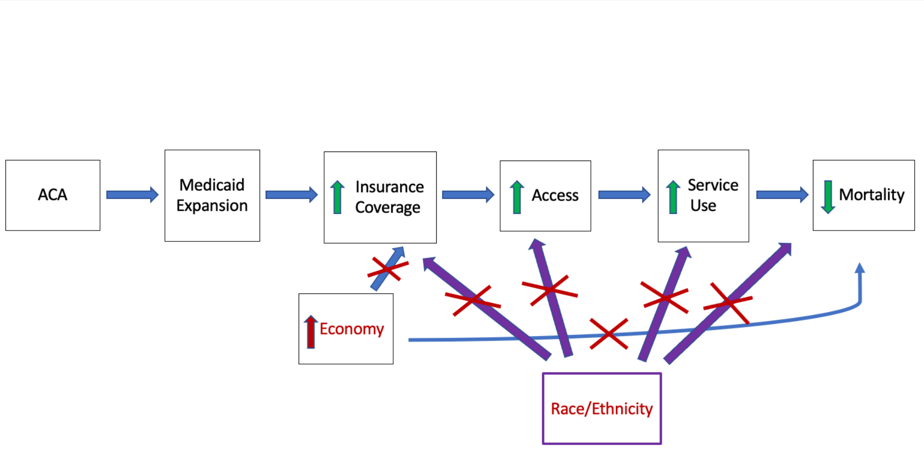

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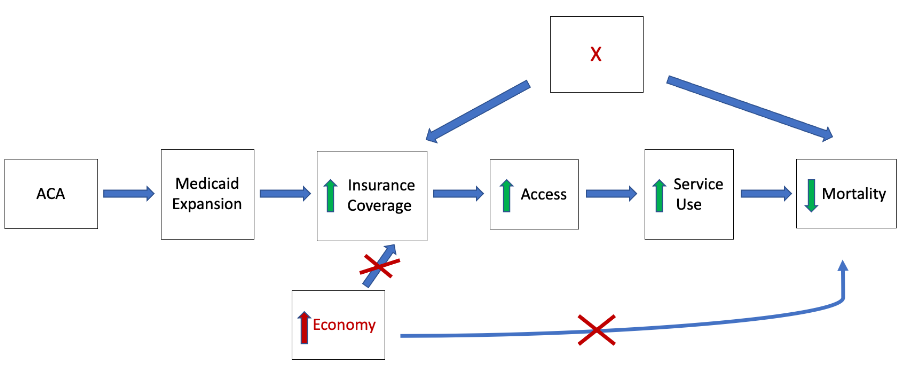

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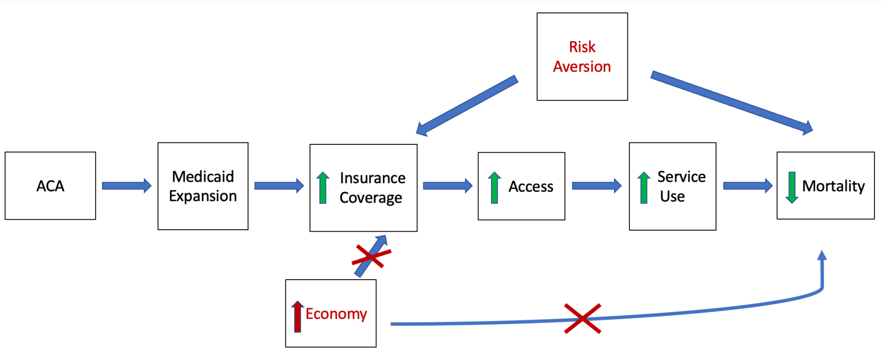

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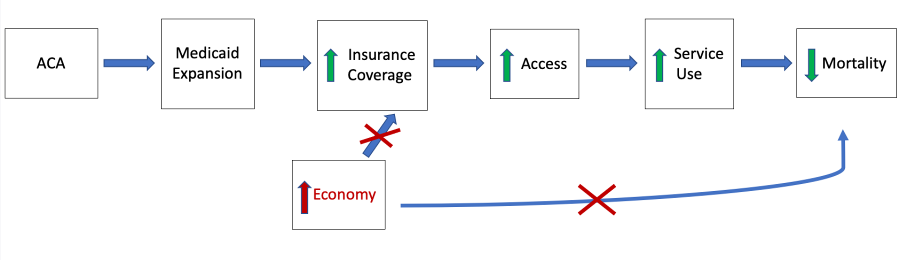

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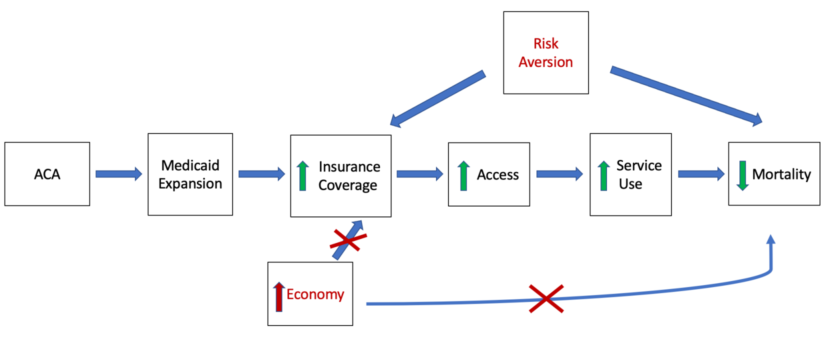

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- Sometimes we can determine which direction that bias is likely to take.
  - Suppose that those who are more risk averse are more likely to sign up for Medicaid coverage when eligible and are more likely to be in better health.
    - Then our estimate of the effect of Medicaid coverage on mortality will be biased *away from zero*.

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  - The goal is to compare statistically identical people who differ only in their exposure to treatment.
  - Under certain conditions, regression can return causal estimates of our relationship of interest.
  - But omitted variable bias is a serious threat when exposure to treatment is non-random.
- So what do we do if randomization is infeasible and we're not confident that regression estimates will be unbiased?
  - Natural experiments and quasi-experimental research design.