

Estimating Treatment Effects - Regression with Matching

April 2, 2026

Estimating Treatment Effects Review

- $ATE = Avg_n[Y^1 - Y^0]$
- $ATE_{est} = Avg_n[Y^1|D = 1] - Avg_n[Y^0|D = 0]$
- When $(Y^1, Y^0) \not\perp D$:

$$ATE_{est} = ATE + \underbrace{\{Avg_n[Y^0|D = 1] - Avg_n[Y^0|D = 0]\}}_{\text{Selection Bias}} \\ + \underbrace{(1 - \pi)(ATT - ATU)}_{\text{Heterogeneous Treatment Effect Bias}}$$

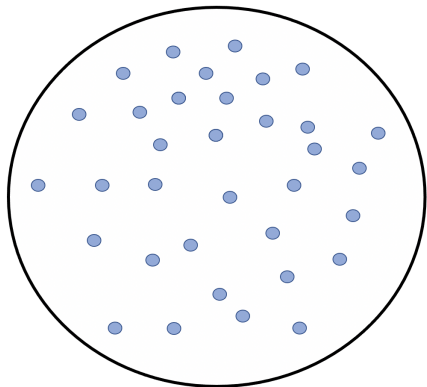
- Randomize?
- $ATE_{est} = \beta_0 + \beta_1 D + \beta_2 X_1 + \beta_3 X_2 + \dots \beta_k X_{k-1} + \varepsilon$

- Can we improve the likelihood that estimating the ATE with non-random assignment returns causal estimates?

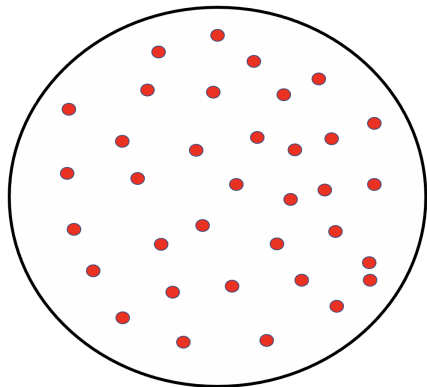
- Can we improve the likelihood that estimating the ATE with non-random assignment returns causal estimates?
- Propensity Score Matching
 - ▶ Intuition: control units that more closely resemble treatment units on *observables* will also more closely resemble treatment units on *unobservables*.

Quasi-Experimental Research Design - Propensity Score Matching

Control Units

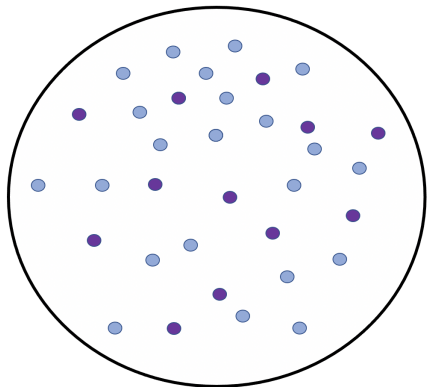


Treatment Units

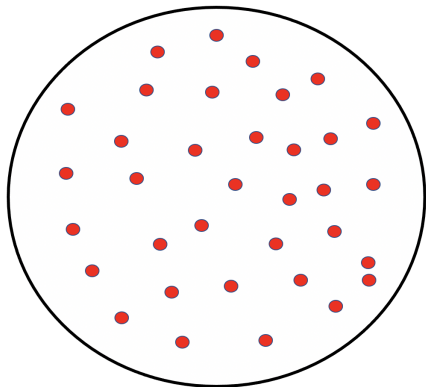


Quasi-Experimental Research Design - Propensity Score Matching

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Treatment Units



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- How similar are observables between the treatment and matched controls?

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5. Compare balance.
 - How similar are observables between the treatment and matched controls?
6. Estimate the ATE.

Quasi-Experimental Research Design - Propensity Score Matching

- Matching Methods

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- ▶ “Closest” is determined by the propensity score.
 - Starting point?
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- ▶ Downsides:
 - Lots of (ad hoc) decision making required.
 - Discard unmatched control units.

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 - ▶ Use propensity scores as regression weights.
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- Remember: regardless of matching technique, PSM assumes that matching on observables removes confounding from unobservables.
 - We have no definitive way to test this assumption.

Quasi-Experimental Research Design - Propensity Score Matching

- Balance Test: *Shau et al. (2018) - Medicaid is Associated with Increased Readmission and Resource Utilization after Primary Total Knee Arthroplasty*

Table 1
Characteristics of the TKA (ICD-9 code 8154) patients from the 2013 NRD.

Risk factors	Before propensity matching				After propensity matching			
	Medicaid, N = 8372	Other insurance, n = 268,261	P Value	Standardized difference	Medicaid, n = 8372	Other insurance, n = 8372	P value ^a	Standardized difference ^b
Age, years (mean ± SD)	56.7 ± 9.4	66.7 ± 9.7	<.0001	1.055	56.7 ± 9.4	56.5 ± 9.6	<.001	0.014
Female sex	6072 (72.5%)	165,227 (61.6%)	<.0001	0.234	6071 (72.5%)	6129 (73.2%)	.004	0.016
Severity of illness (major/extreme loss vs other)	549 (6.6%)	13,685 (5.1%)	<.0001	0.062	547 (6.5%)	477 (5.7%)	<.001	0.035
Discharged to skilled facility	2118 (25.3%)	75,028 (28.0%)	<.0001	0.061	2118 (25.3%)	2142 (25.6%)	.261	0.007
Smoking	2711 (32.4%)	59,997 (22.4%)	<.0001	0.226	2710 (32.4%)	2664 (31.8%)	.038	0.012